

Study of the temperature dependence of single-particle and pair coherent condensate densities for a Bose liquid with a "depleted" single-particle Bose-Einstein condensate (BEC) at $T \neq 0$

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Abstract

We study the temperature dependence of single-particle and pair coherent condensate densities for a Bose liquid with a "depleted" single-particle Bose-Einstein condensate (BEC) at $T \neq 0$. Our investigations were based on the field-theoretical Green's functions. On the basis of the use of empirical data about the speeds of the first and second sounds is described superfluid state at $T \neq 0$. It is studied the structure of superfluid state taking into account appearance with the different from zero temperatures of normal component and taking into account the branch of the second sound, whose speed approaches zero at $T \rightarrow T_\lambda$. The bare interaction between bosons was chosen in the form of a repulsive Azis potential, the Fourier component of which was an oscillatory and sign-varying function of the momentum. The obtained temperature dependence of single-particle and pair coherent condensate and total densities has a good agreement with experiment data.

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1 Introduction

Despite the big progress achieved in the theory of superfluidity since the pioneer works by Landau [1], Bogolyubov [2], Feynman [3] and others [4]–[22], the task of constructing a microscopic theory of a superfluid (SF) state of a ${}^4\text{He}$ Bose liquid cannot be considered complete. Analysis of experimental and theoretical activities concerning with unique superfluidity phenomena investigation has one conclude that microscopic theory of superfluidity is still incomplete. There are some unsolved contradictions between theory and experiment as concerned with superfluid liquid hydrodynamics as with structure of quasiparticle spectrum. These contradictions reduce to the question of quantum-mechanical structure of superfluid ${}^4\text{He}$ component below the λ -point. This question is crucial in creation of consistent microscopic superfluid theory of Bose liquid both for the case of zero temperature and for the case of different from zero temperatures.

In works [23], [24] one introduces at $T = 0$ the microscopic model of the SF state of a Bose liquid with a single-particle Bose-Einstein condensate (BEC) suppressed because of interaction, based upon a renormalized field perturbation theory with combined variables [11]–[14], allows one to obtain a self-consistent “short” system of nonlinear integral equations for the self-energy parts $\tilde{\Sigma}_{ij}(p, \epsilon)$, by means of truncating the infinite series in the small density of the BEC ($n_0/n \ll 1$). By the same token, one can work out a self-consistent microscopic theory of a superfluid Bose liquid and perform an ab initio calculation of the spectrum of elementary excitations $E(p)$, starting from realistic models of pair interaction potential $U(r)$.

In this work we will study of the superfluid state structure of Bose-liquid ${}^4\text{He}$ under nonzero temperature ($0 < T < T_\lambda$). The same task was consider in work [25] on the basic of Chester [26] IBG wave function Ψ_{IBG} by Jastrow factors $F = \prod f_{ij}$. We use the microscopic model proposed in [27] for the superfluid Bose liquid with a “depleted” BEC. The small parameter of these model is the ratio of the density of the BEC to the total density Bose liquid $\rho_0/\rho_s \ll 1$ in contrast to the Bogolyubov theory [2] for a almost ideal Bose gas, where the small parameter is the ratio of the number of overcondensate excitations to the number of particles in the intense BEC ($n - n_0)/n_0 \ll 1$.

Such approach is based on the analysis of the numerous precise experimental data on neutron inelastic scattering [22], [28]–[30] and to results in quantum evaporation of ${}^4\text{He}$ atoms [31], according to which the maximal density ρ_0 of the single-particle BEC in the ${}^4\text{He}$ Bose liquid even at very low temperatures $T \ll T_\lambda$ does not exceed 10% of the total density ρ of liquid ${}^4\text{He}$, whereas the density of the SF component $\rho_s \rightarrow \rho$ at $T \rightarrow 0$ [10]. Such a low density of the BEC is implied by strong interaction between ${}^4\text{He}$ atoms and is an indication of the fact that the quantum structure of the part of the SF condensate in He II carrying the “excess” density $(\rho_s - \rho_0) \gg \rho_0$ calls for a more thorough investigation. So, in this case the density of the superfluid component ρ_s is determined by the quantity of the renormalized anomalous self-energy parts $\tilde{\Sigma}_{12}(0, 0)$, which is a superposition of the “depleted” single-particle BEC and the intense “Cooper” pair coherent condensate (PCC), with coincident phases (sings) of the corresponding order parameters. Such a PCC emerges due to an effective attraction between bosons in some regions of momentum space, which results from an oscillating sign-changing momentum dependence of the Fourier component $V(p)$ of the interaction potentials $U(r)$ with the inflection points in the radial dependence.

The system of Dyson-Belyaev equations [5], which describe the superfluid (SF) state of a Bose liquid with strong interaction between bosons and a weak single-particle Bose-Einstein condensate (BEC) and which allows one to express the normal \tilde{G}_{11} and anomalous \tilde{G}_{12} renormalized single-particle boson Green functions in terms of the respective self-energy parts $\tilde{\Sigma}_{11}$ and $\tilde{\Sigma}_{12}$, has the next analytic form of [27]:

$$\tilde{\Sigma}_{11}(\mathbf{p}, \epsilon) = n_0 \Lambda(\mathbf{p}, \epsilon) \tilde{V}(\mathbf{p}, \epsilon) + n_1 V(0) + \tilde{\Psi}_{11}(\mathbf{p}, \epsilon); \quad (1)$$

$$\tilde{\Sigma}_{12}(\mathbf{p}, \epsilon) = n_0 \Lambda(\mathbf{p}, \epsilon) \tilde{V}(\mathbf{p}, \epsilon) + \tilde{\Psi}_{12}(\mathbf{p}, \epsilon), \quad (2)$$

where

$$\tilde{\Psi}_{ij}(\vec{p}, \epsilon) = i \int \frac{d^3 \vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} G_{ij}(\vec{k}) \tilde{V}(\vec{p} - \vec{k}, \epsilon - \omega) \Gamma(\vec{p}, \epsilon, \vec{k}, \omega) \quad (3)$$

$$\tilde{V}(\vec{p}, \epsilon) = V(\vec{p}) [1 - V(\vec{p})\Pi(\vec{p}, \epsilon)]^{-1}. \quad (4)$$

where $V(p)$ is the Fourier component of the bare potential of the pair interaction of the bosons; $\tilde{V}(\vec{p}, \epsilon)$ is the renormalized (screened) pair interaction between bosons because of many-particle collective effects; $\Pi(\vec{p}, \epsilon)$ is the boson's polarization operator which describe many-particle correlation effects

$$\begin{aligned} \Pi(\mathbf{p}, \epsilon) = i \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \Gamma(\mathbf{p}, \epsilon, \mathbf{k}, \omega) \{ & G_{11}(\mathbf{k}, \omega) \\ & \times G_{11}(\mathbf{k} + \mathbf{p}, \epsilon + \omega) + G_{12}(\mathbf{k}, \omega)G_{12}(\mathbf{k} + \mathbf{p}, \epsilon + \omega) \} ; \end{aligned} \quad (5)$$

where $\Gamma(\vec{p}, \epsilon; \vec{k}, \omega)$ is the vertex part (three-pole) describing many-particles correlations of the local field's type; $\Lambda(\vec{p}, \epsilon) = \Gamma(\vec{p}, \epsilon, 0, 0) = \Gamma(0, 0, \vec{p}, \epsilon)$ is the vertex part with zero values of input momentum and energy; and n_0 is the number particles in BEC, $n_1 = n - n_0$ is the number of overcondensate particles ($n_1 \gg n_0$), which is determined by the condition of the total particle number conservation:

$$n = n_0 + n_1 = n_0 + i \int \frac{d^3 \vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} G_{11}(\vec{k}, \omega). \quad (6)$$

Take into account the residues at the poles of single-particle Green functions $\tilde{G}_{ij}(\mathbf{p}, \epsilon)$, neglecting the contributions of eventual poles of the functions $\Gamma(\mathbf{p}, \epsilon, \mathbf{k}, \omega)$ and $\tilde{V}(\mathbf{p}, \epsilon)$, which do not coincide with the poles of $\tilde{G}_{ij}(\mathbf{p}, \epsilon)$, the functions $\tilde{\Psi}_{ij}(\mathbf{p}, \epsilon)$ on the “mass shell” $\epsilon = E(p)$ assume the following form (at $T = 0$):

$$\begin{aligned} \tilde{\Psi}_{11}(\mathbf{p}, E(p)) = \frac{1}{2} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \Gamma(\mathbf{p}, E(p); \mathbf{k}, E(k)) \\ \times \tilde{V}(\mathbf{p} - \mathbf{k}, E(p) - E(k)) \left[\frac{A(\mathbf{k}, E(k))}{E(k)} - 1 \right] ; \end{aligned} \quad (7)$$

$$\begin{aligned} \tilde{\Psi}_{12}(\mathbf{p}, E(p)) = -\frac{1}{2} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \\ \times \tilde{V}(\mathbf{p} - \mathbf{k}, E(p) - E(k)) \Gamma(\mathbf{p}, E(p); \mathbf{k}, E(k)) \\ \times \frac{n_0 \Lambda(\mathbf{k}, E(k)) \tilde{V}(\mathbf{k}, E(k)) + \tilde{\Psi}_{12}(\mathbf{k}, E(k))}{E(k)} , \end{aligned} \quad (8)$$

where

$$A(\vec{p}) = n_0 \Lambda(\vec{p}, E(\vec{p})) \tilde{V}(\vec{p}, E(\vec{p})) + n_1 V(0) + \tilde{\Psi}_{11}(\vec{p}) + \frac{\vec{p}^2}{2m} - \mu, \quad (9)$$

here μ is the chemical potential of the quasiparticles, which satisfies the Hugengoltz-Pines relation [6]

$$\mu = \tilde{\Sigma}_{11}(0, 0) - \tilde{\Sigma}_{12}(0, 0) . \quad (10)$$

The quasiparticle spectrum corresponding to the poles of Green's function is determined in general, with allowance for relations (1,2), by the next expression

$$E(\vec{p}) = \sqrt{A^2(\vec{p}) - \left[n_0 \Lambda(\vec{p}, E(\vec{p})) \tilde{V}(\vec{p}, E(\vec{p})) + \tilde{\Psi}_{12}(\vec{p}) \right]^2} . \quad (11)$$

2 The structure of the superfluid Bose liquid state at $T \neq 0$

The main purpose of our research is the investigation of superfluid helium condensate structure by means of finding temperature dependence of single-particle and pair coherent condensate densities under assumption that superfluid component is a superposition of a weak single-particle BEC and an intensive PCC.

Let's consider the superfluid bose liquid state at $T \neq 0$ when there are superfluid component density $\rho_s(T)$ and normal component density $\rho_n(T)$ simultaneously. As it was shown in [13, 14] the expression for the renormalized Green's function $\tilde{G}_{ij}(p)$ is constructed with help of combined variables:

$$\tilde{\Psi}(x) = \tilde{\Psi}_L(x) + \tilde{\Psi}_{sh}(x). \quad (12)$$

In the long-wavelength region $|\vec{k}| < k_0$ (where k is some characteristic momentum) this variables are just the hydrodynamics variables $\tilde{\Psi}_L(x)$ in the spirit of Landau quantum hydrodynamics [1], and in the short-wavelength region $|\vec{k}| > k_0$ they coincide with the usual field operators $\tilde{\Psi}_{sh}(x)$:

$$\begin{aligned} \tilde{\Psi}_L(x) &= \sqrt{\langle \tilde{n}_L \rangle} \left[1 + \frac{\tilde{n}_L - \langle \tilde{n}_L \rangle}{2\langle \tilde{n}_L \rangle} + i\tilde{\phi}_L \right]; \quad \tilde{\Psi}_{sh} = \psi_{sh} e^{-i\tilde{\phi}_L}; \\ \psi_{sh} &= \psi - \psi_L; \quad \psi_L(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{|\mathbf{k}| < k_0} a_{\mathbf{k}} e^{i\mathbf{k}\mathbf{r}} = \sqrt{\langle \tilde{n}_L \rangle} e^{i\tilde{\phi}_L}. \end{aligned} \quad (13)$$

Such an approach means that the separation of the Bose system into a macroscopic coherent condensate and a gas of supracondensate excitations is made not on the statistical level, like in the case of a weakly nonideal Bose gas [2], but on the level of ab initio field operators, which are used to construct a microscopic theory of the Bose liquid.

In the region of small $p \neq 0$ at $T \rightarrow 0$ Green's functions have the form

$$\begin{aligned} \tilde{G}_{11}(p) &= -n_s g_{\varphi\varphi}(p) - ig_{\varphi\pi}(p) - \\ &- \frac{1}{4\rho_s} g_{\pi\pi}(p) - \frac{n_s}{2} \Phi_{\varphi\varphi}(p) \dots \end{aligned} \quad (14)$$

$$\tilde{G}_{12}(p) = n_s g_{\varphi\varphi}(p) - \frac{1}{4\rho_s} g_{\pi\pi}(p) - \frac{n_s}{2} \Phi_{\varphi\varphi}(p) \dots \quad (15)$$

where

$$\begin{aligned} \Phi_{\varphi\varphi}(p) &= \int_{|q| < q_0} \frac{d^4 q}{2\pi^4} g_{\varphi\varphi}(q) g_{\varphi\varphi}(p - q), \\ p &= (\vec{k}, \epsilon), q = (\vec{q}, \omega) \end{aligned} \quad (16)$$

where $g_{\mu\nu}(p)$ are the "hydrodynamics" Green's functions, which are associated with the long-wavelength fluctuations of the phase and density of the condensate ($\mu\nu = \varphi\pi$). The expressions for $g_{\varphi\varphi}(p)$, $g_{\varphi\pi}(p)$, $g_{\pi\pi}(p)$ was calculated in [12] for $T > 0$. It contains sums of two pole terms, corresponding to first and second sound with velocities c_1 and c_2 in the Bose liquid with the normal and superfluid components:

$$\begin{aligned} g_{\mu\nu}(k, \epsilon) &= \frac{(a_{\mu\nu} - d_{\mu\nu} \rho_n / \rho)}{\epsilon^2 - c_1^2 k^2} + \frac{b_{\mu\nu} \rho_n / \rho}{\epsilon^2 - c_2^2 k^2} \\ \mu, \nu &= \varphi, \pi \end{aligned} \quad (17)$$

where $\rho = \rho_n + \rho_s$ is the total density of the liquid, and the coefficients $a_{\mu\nu}$, $d_{\mu\nu}$ and $b_{\mu\nu}$ are independent of T at low temperatures. This result establishes a unique correspondence between the microscopic field theory of superfluidity [5, 7] and the macroscopic two-fluid hydrodynamics [8, 10].

It follows from Eqs. (14), (15) and (17) that the pole parts of the renormalized Green's functions \tilde{G}_{ij} can be represented in the form

$$\tilde{G}_{ij}(\vec{k}, \epsilon) = \frac{(A_{ij} - D_{ij} \rho_n / \rho)}{\epsilon^2 - c_1^2 k^2} + \frac{B_{ij} \rho_n / \rho}{\epsilon^2 - c_2^2 k^2} \quad i, j = 1, 2 \quad (18)$$

We henceforth assume that expression (18) is valid in the entire temperature interval $T < T_\lambda$. It follows from Eqs. (18) that for $T \rightarrow 0$, where $\rho_n \rightarrow 0$, the leading contribution to the integral

over energy ϵ in (3) comes from the first-sound pole $\epsilon = c_1 k$ of the Green's functions. However, at higher temperatures $T > 1K$, where $\rho_n \sim \rho_s$, the main part, because of the strong inequality $c_1 \gg c_2$, is played by the low-energy pole $\epsilon = c_2 k$ which corresponds to second sound.

At finite temperatures ($T \neq 0$), after taking into account the contributions of the first and second poles of the Green's functions (18), we obtain for the self-energy parts in $\Gamma = \Lambda = 1.5$ approximation

$$\begin{aligned} \tilde{\Sigma}_{ij}(\vec{k}, T) = & -\frac{1}{2} \int \frac{d^3 \vec{q}}{(2\pi)^3} \tilde{V}(\vec{k} - \vec{q}) \left\{ \left[A_{ij} - D_{ij} \frac{\rho_n(T)}{\rho} \right] \frac{1}{c_1 q} \right. \\ & \left. \times \coth \left(\frac{c_1 q}{2T} \right) + B_{ij} \frac{\rho_n(T)}{\rho} \frac{1}{c_2 q} \coth \left(\frac{c_1 q}{2T} \right) \right\} \end{aligned} \quad (19)$$

It should be emphasized that the long-wavelength approximation for the Green's functions (18) in this case are valid because of the divergence of the temperature factor $\coth(c_2 q/2T)$ at $q \rightarrow 0$ and the rather rapid decay of the interaction kernel ($q \rightarrow \infty$). Moreover, the system of equations (19) does not need to be matched with the expression for the renormalized quasi-particle spectrum $E(k)$, as it is ordinarily done in microscopic field theory for $T \rightarrow 0$. It's true because the substitution of the empirical spectra of the first and second sound (with the experimental values of the velocities c_1 and c_2) into the expressions for the Green's functions $\tilde{G}_{ij}(p)$ corresponds to automatically taking all the necessary renormalizations into account.

Using (19), one can determine the superfluid order parameter for $T \neq 0$:

$$\tilde{\Sigma}_{12}(0, T) = \Psi_0(T) + \Psi_s(T) \frac{\rho_s(T)}{\rho}, \quad (20)$$

where

$$\Psi_0(T) = -\frac{1}{2} \int \frac{d^3 \vec{q}}{(2\pi)^3} \tilde{V}(q) \left[\frac{A_{12} - D_{12}}{c_1 q} \coth \left(\frac{c_1 q}{2T} \right) + \frac{B_{12}}{c_2 q} \coth \left(\frac{c_2 q}{2T} \right) \right], \quad (21)$$

$$\Psi_s(T) = -\frac{1}{2} \int \frac{d^3 \vec{q}}{(2\pi)^3} \tilde{V}(q) \left[\frac{D_{12}}{c_1 q} \coth \left(\frac{c_1 q}{2T} \right) - \frac{B_{12}}{c_2 q} \coth \left(\frac{c_2 q}{2T} \right) \right], \quad (22)$$

Since under $T = 0$ the density of the superfluid component ρ_s coincides with the total mass density of the Bose liquid $\rho = mn$ and is proportionate to $\Sigma_{12}(0, 0)$ (it plays a part of superfluid order parameter) we can write the following expression

$$\rho_s = \rho_0 + \tilde{\rho}_s = \beta m \frac{\Sigma_{12}(0)}{\Lambda(0) \tilde{V}(0)} = \beta m [n_0(1 - \gamma) + \theta], \quad (23)$$

where

$$\gamma \equiv \frac{1}{(2\pi)^2 \Lambda(0) \tilde{V}(0)} \int_0^\infty \frac{k^2 dk}{E(k)} \left(\Lambda(k) \tilde{V}(k) \right)^2, \quad (24)$$

$$\theta = -\frac{1}{(2\pi)^2 \Lambda(0) \tilde{V}(0)} \int_0^\infty \frac{k^2 dk}{E(k)} \Lambda(k) \tilde{V}(k) \Psi(k), \quad (25)$$

where β is the constant to be identified. Starting from the definition of the single-particle BEC density ρ_0 as $\rho_0 = mn_0$, we obtain $\beta = (1 - \gamma)^{-1}$. Thus the "Cooper" PCC density takes form:

$$\tilde{\rho}_s = mn_1 = \frac{m\theta}{1 - \gamma}, \quad (26)$$

where $\rho_0 = mn_0$ is the density of the single-particle BEC and $\tilde{\rho}_s$ is the density of "Cooper" PCC.

On the other hand, the temperature dependence of BEC density $\rho_0(T) = mn_0(T)$ is due to entire particle number conservation condition (5) will be determined the following expression

$$\frac{\rho_0(T)}{\rho} = 1 - \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left\{ \left[A_{11} - D_{11} \frac{\rho_n(T)}{\rho} \right] \frac{1}{c_1 q} \right. \\ \left. \times \coth \left(\frac{c_1 q}{2T} \right) + B_{11} \frac{\rho_n(T)}{\rho} \frac{1}{c_2 q} \coth \left(\frac{c_2 q}{2T} \right) \right\}. \quad (27)$$

The density of pair coherent condensate is

$$\frac{\rho_s(T)}{\rho} = \frac{\Psi_0(T)}{\tilde{V}(0)n} \left[1 - \frac{\Psi_s(T)}{\tilde{V}(0)n} \right]^{-1}. \quad (28)$$

Thus the "Cooper" PCC density at $T = 0$ in our model can be found in two independent ways. We can find it directly with help of expression (26) or we can subtract (27) from (28) and then find result at $T = 0$. Thus we can verify the self-consistency of the proposed model via comparison the results of independent calculations of the PCC density of superfluid component for the zero temperature.

3 The iterative scheme of calculation and results

In order to calculate the the temperature dependence of single-particle and pair coherent condensate densities within the model of a Bose liquid with a suppressed BEC considered, at first, we have to find the Fourier component of the bare potential of the pair interaction of the bosons and then, to find the renormalized (screened) pair interaction between bosons, which is determined (4), so we must to calculate the boson polarization operator (5).

The pair interaction between bosons was chosen in the form of a regularized repulsive potential in the the Aziz potential [32, 33]:

$$U_A(r) = \begin{cases} A \exp(-\alpha r - \beta r^2) - \exp[-(r_0/r - 1)^2] \sum_{k=0}^2 c_{2k+6} r^{-2k-6}, & r < r_0 \\ A \exp(-\alpha r - \beta r^2) - \sum_{k=0}^2 c_{2k+6} r^{-2k-6}, & r \geq r_0 \end{cases} \quad (29)$$

where $A = 1.8443101 \times 10^5$ K, $\alpha = 10.43329537 \text{ \AA}^{-1}$, $\beta = 2.27965105 \text{ \AA}^{-2}$, $c_6 = 1.36745214 \text{ K} \times \text{\AA}^6$, $c_8 = 0.42123807 \text{ K} \times \text{\AA}^8$, $c_{10} = 0.17473318 \text{ K} \times \text{\AA}^{10}$. Such potential remain finite at $r = 0$ due to the nonanalytic exponential dependence on r , which suppresses any power divergence at $r \rightarrow 0$. It has the Fourier component $V(p)$, which is an oscillatory and sign-varying function of the momentum p as a result of the "excluded volume" effect and the quantum diffraction of the particles on one another.

Then, in order to calculate the polarization operator, using Eqs. (7) and (8), a numerical calculation in the first approximation of the functions $\Phi_1(p) \equiv \tilde{\Psi}_{11}(\mathbf{p}, E_0(\mathbf{p}))$ and $\Psi_1(p) \equiv \tilde{\Psi}_{12}(\mathbf{p}, E_0(\mathbf{p}))$, was conducted. For the zeroth approximation, the "screened" potential (4) were taken:

$$\tilde{V}_0(p) = \frac{V_0 j_1(pa)}{pa - V_0 \Pi_0 j_1(pa)} \quad (30)$$

at some constant negative value of Π_0 . Then, using the functions $\Phi_1(p)$ and $\Psi_1(p)$ obtained, the first approximation for the polarization operator $\Pi_1(p)$ was calculated, using Eqs. (14)–(16), (18) at $\Gamma = 1$. The limiting value $\Pi_1(0)$ was compared with the exact thermodynamic value of the polarization operator of the ^4He Bose liquid at $p = 0$ and $\omega = 0$, which determines the compressibility of the Bose system [13]: $\Pi(0, 0) = -\frac{n}{mc_1^2}$.

The absolute value $|\Pi(0, 0)|$ turned out to be almost 1.5 times greater than the calculated value $|\Pi_1(0)|$. This provides for an estimate of the vertex Γ at $p = 0$ in the first approximation as

$\Gamma_1 \equiv \Lambda_1 \simeq 1.5$. The second approximation $\Phi_2(p)$ and $\Psi_2(p)$ was obtained from Eqs. (7), (8) with the constant value $\Gamma_1 \equiv \Lambda_1$ and the first approximation for the renormalized pseudopotential, Eq. (4):

$$\tilde{V}_1(p) = \frac{V_0 j_1(pa)}{pa - V_0 \Pi_1(p) j_1(pa)} . \quad (31)$$

Such an iterative procedure was repeated four to six times and used to improve precision in the calculation of the polarization operator. At each stage, equations (9) and (11) were used to reproduce the quasiparticle spectrum $E(p)$, and the rate of convergence of the iterations was watched, as well as the degree of proximity of $E(p)$ to the empirical spectrum $E_{\text{exp}}(p)$. The fitting parameter in these calculations was the amplitude V_0 of the seed potential (29) at the value of $a = 2.44 \text{ \AA}$, which is equal to twice the quantum radius of the ${}^4\text{He}$ atom. The BEC density, in accordance with the experimental data [28]–[31], was fixed at $n_0 = 9\%n = 1.95 \cdot 10^{21} \text{ cm}^{-3}$.

For the numerical computation of temperature dependencies of single-particle and pair coherent condensate density we have to take into account that the first (hydrodynamics) sound velocity is practically independent of T and in the given approximation can be determined as $c_1 = [\tilde{V}(0)n/m^*]^{1/2}$ (here m^* is the effective mass of quasiparticles), whereas the velocity of second sound c_2 is substantially T -dependent, varying from $c_2(0) = c_1/\sqrt{3}$ at $T = 0$ to a value $c_2(T) \simeq 20 \text{ m/s}$ in the region $T > 1K$ [8]. At $T \rightarrow T_\lambda$ the velocity $c_2 \rightarrow 0$. Thus as the λ point is approached the main part, owing to the strong inequality $c_1 \gg c_2$, begins to be played by the last terms in the integrands in (21), (27), which are proportional to B_{12} and B_{11} and contain the temperature factor

$$f(q, T) = \frac{1}{c_2(T)q} \coth \left(\frac{c_2(T)q}{2k_B T} \right) = \frac{2T}{c_2^2(T)q^2} , \quad c_2q < T \quad (32)$$

which diverges quadratically as $q \rightarrow 0$. At the same time the width of the singular peak increases rapidly with increasing T and decreasing c_2 . (the momentum dependence of temperature factor, obtained for different temperatures, is depicted in fig. 1). As a result, with increasing T there is an increase of the contribution to the integral (21) from the repulsive part of the potential $\tilde{V}(q) > 0$ in the long-wavelength region $q < \pi/a$ and a decrease of the function $\Psi_0(T)$. The function $\Psi_0(T)$ plays a part of the superfluid order parameter and is positive at low $T < c_2 q$ owing to the strong attraction $\tilde{V}(q) < 0$ in the region $\pi/a < q < 2\pi/a$. At a certain critical temperature $T = T_c$ the function $\Psi_0(T)$ goes to zero and then becomes negative (for $T > T_c$), that corresponds to destruction of the superfluid state ($\rho_s = 0$), i.e. T_c coincides with the λ point.

In a similar way with increasing T , there is increase of the negative contribution to the integral (27) and decrease of $\rho_0(T)$ until at a certain point $T = T_0$ the density of the BEC vanishes and formally becomes negative for $T > T_0$.

The results of the numerical calculations of the temperature dependence of the densities of single-particle and paired parts of superfluid density in the range of temperatures from zero to the point of lambda-transition, are depicted on fig.2.

4 Conclusions

Thus, on the basis of the use of empirical data about the speeds of the first and second sounds in this work is described superfluid state at $T \neq 0$. It is studied the structure of superfluid state taking into account appearance with the different from zero temperatures of normal component and taking into account the branch of the second sound, whose speed approaches zero at $T \rightarrow T_\lambda$. We obtained the analytic expression (27, 28) for computation ρ_0 and ρ_s densities. and constructed and realized the numerical scheme for computation of the temperature dependence of single-particle and pair coherent condensate densities. The temperature dependence of the densities of single-particle condensate and total density of superfluid component are obtained on the basis of the microscopic model of superfluid state of Bose-liquid with the depressed single-particle BEC in the range from $T = 0$ to temperatures close to the environment of point of λ -transition.

Note also that the self-consistency of this model is corroborated by the fact that the theoretical value of total particle density calculated from Eq. (6), $n_{\text{th}} = 2.1 \cdot 10^{22} \text{ cm}^{-3}$, is quite close to the experimental value of the density of particles in liquid ^4He , $n = 2.17 \cdot 10^{22} \text{ cm}^{-3}$ (at $n_0 = 9\% n$). On the other hand, the density n_1 of supracondensate particles, calculated from Eqs. (26) at $T = 0$ at the values of the parameters indicated, is about $0.93 n$, which is also in good accordance with experiment, taking into account that the BEC density is determined up to $\pm 0.01 n$.

It goes without saying, the given results, which correspond to the approximation of self-consistent field, cannot be used directly near the λ -point, where thermodynamic fluctuations play the key role. But the dependences, depicted in fig.2, qualitatively correctly describe the temperature dependence of the density of superfluid component.

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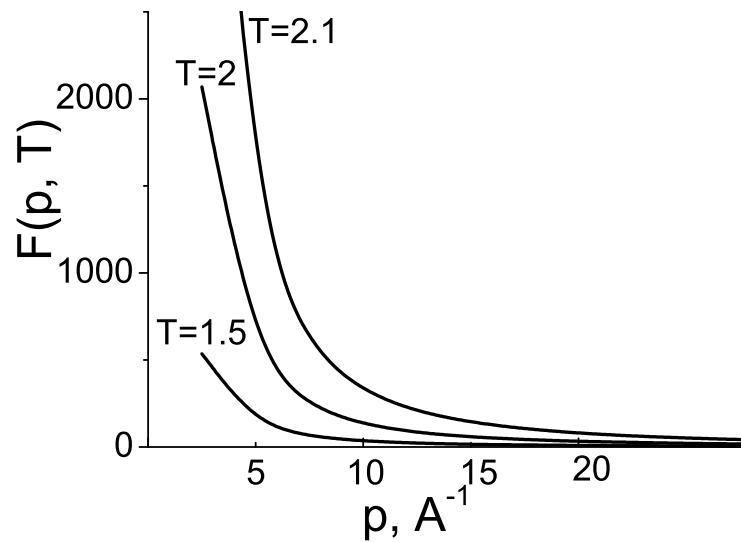


Fig. 1. The momentum dependence of the temperature factor (32), obtained for different temperatures.

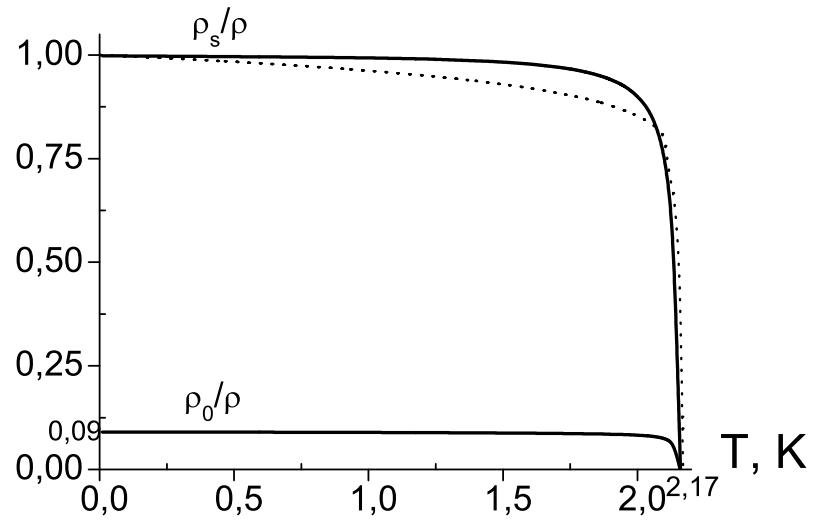


Fig. 2. The temperature dependence of the density of BEC (lower curve) and total density of superfluid component (upper curve), obtained according (27) and (28) for next parameters $A_{11} = 6.14K$, $D_{11} = 2.03K$, $B_{11} = 0.00018K$, $A_{12} = 6.21K$, $D_{12} = 3.12K$, $B_{12} = 0.00225K$. The empirical temperature dependence of of superfluid component [8] (circles).